In the Polygorial paper, I assert that the ratio of $\mathcal{P}_n^6$ to $\mathcal{P}_n^3$ produces the Catalan number C_n (A000108$_n$).

Proof. Given

$$\mathcal{P}_n^k \equiv \frac{n!}{2^n} (k-2)^n \left(\frac{2}{k-2}\right)_n \tag{1}$$

$$C_n \equiv \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!} \tag{2}$$

then[1]

$$\begin{aligned}
\mathcal{P}_n^6 &\equiv \frac{n!}{2^n}4^n \left(\frac{1}{2}\right)_n \\
&= n!2^n \frac{\Gamma\left(\frac{1}{2}+n\right)}{\Gamma\left(\frac{1}{2}\right)} \\
&= n!2^n \frac{\Gamma\left(\frac{1}{2}+n\right)}{\sqrt{\pi}} \\
&= n!2^n \frac{(2n-1)!!}{2^n}\frac{\sqrt{\pi}}{\sqrt{\pi}} \\
&= n!(2n-1)!! \\
&= n!\frac{(2n)!}{2^n n!} \\
&= \frac{(2n)!}{2^n}
\end{aligned} \tag{3}$$

$$\begin{aligned}
\mathcal{P}_n^3 &\equiv \frac{n!}{2^n}(2)_n \\
&= \frac{n!}{2^n}\frac{\Gamma(2+n)}{\Gamma(2)} \\
&= \frac{n!^2(n+1)}{2^n}
\end{aligned} \tag{4}$$

thus

$$\frac{\mathcal{P}_n^6}{\mathcal{P}_n^3} = \frac{\frac{(2n)!}{2^n}}{\frac{n!^2(n+1)}{2^n}} = \frac{(2n)!}{(n+1)!n!} = C_n$$

□

[1] Mathworld's "Double factorial article" cites G Arfken, *Mathematical Methods for Physicists*, 3e, 1985, p. 548, for $\Gamma\left(\frac{1}{2}+n\right) = \frac{(2n-1)!!}{2^n}\sqrt{\pi}$ and $(2n-1)!! = \frac{(2n)!}{2^n n!}$